EECS 442 Discussion

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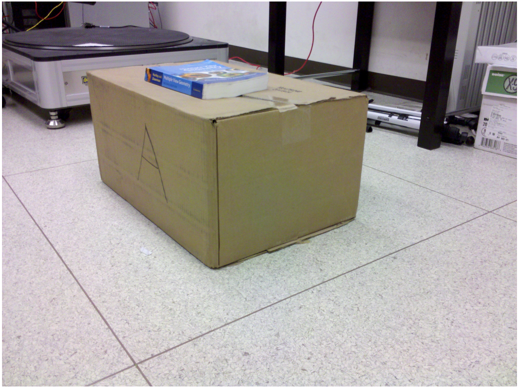
Announcements

- HW3 due 10/29

Single View Geometry: Vanishing Points

- Goal: Find the camera calibration matrix K given a single image by using vanishing points



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- How can we relate their vanishing points v_i and v_j?
  $\left(K^{-1}v_i\right)^\top \left(K^{-1}v_j\right) = 0$
- Consider:

$$\left(v_i^\top K^{-\top}\right)\left(K^{-1}v_j\right) = 0$$

$$v_i^\top \underbrace{\left(K^{-\top}K^{-1}\right)}_{W} v_j = 0$$

Image of the Absolute Conic

$$v_i^\top W v_j = 0$$

- W is "the image of the absolute conic" (more detail in [HZ] 8.5)
- If we can solve for W, we can recover K

How do we solve for W?

$$W = \begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}$$

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- $W = K^{-\top}K^{-1}$ is symmetric

How do we solve for W?

$$W = \begin{bmatrix} w_1 & 0 & w_2 \\ 0 & w_1 & w_3 \\ w_2 & w_3 & w_4 \end{bmatrix}$$

- $W = K^{-\top}K^{-1}$ is symmetric
- Assume camera is zero-skew ($s = 0$) and square pixels ($\alpha_x = \alpha_y$)

How do we solve for W?

$$v_i^\top W v_j = 0$$

$$\begin{bmatrix} v_{ix} & v_{iy} & 1 \end{bmatrix} \begin{bmatrix} w_1 & 0 & w_2 \\ 0 & w_1 & w_3 \\ w_2 & w_3 & w_4 \end{bmatrix} \begin{bmatrix} v_{jx} \\ v_{jy} \\ 1 \end{bmatrix} = 0$$

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$$\begin{bmatrix} v_{ix}v_{jx} + v_{iy}v_{jy} & v_{ix} + v_{jx} & v_{iy} + v_{jy} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = 0$$

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$$\underbrace{\begin{bmatrix} v_{ix}v_{jx} + v_{iy}v_{jy} & v_{ix} + v_{jx} & v_{iy} + v_{jy} & 1 \end{bmatrix}}_{A_{ij}} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = 0$$

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- How many pairs of orthogonal vanishing points do we need?

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Given W, how do we solve for K?

- Recall that $W = K^{-\top}K^{-1}$
- For a symmetric matrix, $A = LL^{\top}$ is called the Cholesky decomposition
- We can find K by finding the Cholesky decomposition of W and then taking the inverse (in MATLAB, we can use the `chol` function)
- Remember to normalize K such that $K_{33} = 1$

MATLAB Exercise

- Go to CTools → Resources → Discussion → 10_28_matlab.zip
- Given a single image, find a set of vanishing points and compute the camera calibration matrix

$$K = \begin{bmatrix} 2493 & 0 & 1023.5 \\ 0 & 2493 & 664 \\ 0 & 0 & 1 \end{bmatrix}$$